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# Gauge boson families in grand unified theories of fermion masses: $E_6^4 \rtimes S_4$

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## Abstract

In third quantization the origin of fermion families is easy to understand: the electron field, the muon field and the tau field are identical fields in precisely the same sense as three electrons are identical and undistinguishable particles of a theory of second quantization. In both cases, the permutation of these fields or particles leaves the lagrangian invariant. One can also extend the concept of family to gauge bosons. This can be obtained through the semidirect product of the gauge group with the group of permutations of  $n$  objects. In this paper we have studied the group  $E_6^4 \rtimes S_4$ . We explain why we have chosen  $E_6$  as fundamental gauge group factor and why we start with a model with four gauge boson/fermion families to accommodate and to fit the standard model with only three fermion families. We suggest a possible symmetry breaking pattern of  $E_6^4 \rtimes S_4$  that could explain quark, lepton and neutrino masses and mixings.

## Introduction

Recent neutrino experiments [1] have shown oscillations among neutrinos of different flavor. This is due to the fact that these neutrinos have masses

and mixing angles between different generations. Differently from the quark sector, where all mixing angles are small, in neutrino oscillations some angles are large or even maximal, like in atmospheric neutrinos. Since in grand unified theories there are stringent relations due to the grand unified symmetry between the quark sector and the lepton sector, it is necessary to understand quark/lepton differences within grand unified models. For instance, in the simplest SU(5) models we find that the down quark mass matrix is the transpose of the charged lepton mass matrix. In SO(10) the up quark Yukawa matrix is equal to the Dirac matrix of the neutrinos.

Several recent works have found interesting explanations of the amazing experimental results on neutrino oscillations, extending the standard model with new discrete flavor symmetries in the lepton sector [3, 4, 5, 6, 7, 8, 9, 10]. The advantage of the discrete symmetries with respect to the continuous one, is that they can naturally explain the peculiar structure of large mixings with a relatively large hierarchy between atmospheric and solar neutrino masses. For instance, a neutrino mass matrix that is almost exactly invariant under the permutations of the two heaviest families, that is a permutation of  $\tau$  neutrino with the  $\mu$  neutrino, directly predicts a maximal atmospheric angle and  $\theta_{13} \simeq 0$ . In perfect agreement with the experimental data. In fact if  $P$  is the operator that permutes the two heaviest families and  $P$  commutes with the mass matrix, then it exists an eigenvector of  $P$   $(0, -1/\sqrt{2}, 1/\sqrt{2})$ , with eigenvalue -1, and this is also a mass eigenstate. If, for instance, this is the heaviest neutrino state then  $\theta_{13} = 0$  and the atmospheric angle is maximal. In a recent paper [5] we have shown that including such a  $S_2$  symmetry in a larger discrete symmetry  $S_3$  that permutes all three families, also the solar mixing angle can be explained in a natural way. It is important to note that such a discrete symmetry directly comes from an extension of field theory, that gives an explanation of the hierarchy between the unification scale and the electroweak scale, this is the so-called third quantization [2].

The difficulty that we have mentioned above is how to embed such a  $S_3$  symmetry in a grand unified model, being compatible with the experimental observation of a strong hierarchy between physical quark masses and small mixing angles.

Several authors are studying this problem, but some questions are still open, and in some cases a fine-tuning is required to make leptons and quark results compatible.

In this paper we make a step forward in the grand unified model building

with respect a previous work [6]. In fact we give a more convincing explanation of the reason why the  $S_3$  singlet is the lightest among the left-handed neutrinos and also why there is so large splitting between the two components of the  $S_3$  doublet. In addition we ameliorate the study of the  $E_6$  unified group [12] embedding it in  $E_6^4 \rtimes S_4$ , and giving a symmetry breaking pattern that simultaneously explains quark and lepton masses and mixings.

At the beginning we will discuss which kind of mass operators that break the  $S_3$  symmetry are necessary in order to explain the quark mass and mixings. To do this we will make use of a montecarlo statistical analysis of the allowed mass operators, the method and some results can be found in [4]. Secondly we will extend our analysis to the charged lepton sector, putting in evidence the difficulties due to the grand unification hypothesis and how to overcome them. Next we propose a model of unification  $E_6^4 \rtimes S_4$  that is the semidirect product of the gauge group  $E_6^4$  with the flavor group  $S_4$  [10], and one possible breaking pattern. Note that the gauge group  $E_6^4$  does not commute with  $S_4$ , but it is the normal subgroup of the group  $G$  obtained as the semidirect product of  $E_6^4$  with  $S_4$ , the permutation symmetry of four families. In such a model the concept of family is extended also to gauge bosons. Each family is composed of a subset of fermions and their own gauge bosons that mediate interactions between the fermion of one family only. Namely, in our specific model, the first family contains 27 fermions (the smallest  $E_6$  irreducible representation) and 78 gauge bosons (the vector boson of  $E_6$ ). These bosons only interact with the 27 fermions of the first family, while fermions of the other families are neutral (with respect them). Similarly the 78 gauge bosons of the second family (corresponding to the second  $E_6$  factor in  $E_6^4$ ) only interacts with the fermions of the second family. In other words the  $S_4$  symmetry group permutes the four families, thus it permutes not only the fermions of the four families but also their gauge bosons. This is another way to understand why  $S_4$  does not commute with  $E_6^4$ , and why  $G$  is the semidirect product (and not direct) between  $E_6^4$  and  $S_4$ . At the end we will see how this  $G$  group can be broken in order to explain in a natural way masses and mixings both of quarks and leptons.

## Quark masses

Before discussing grand unified models we want to understand how to break  $S_3$  in the quark sector, namely which operators we need to introduce in or-

der to explain quark masses and mixings. If we want to extend the standard model in a theory of third quantization, that explains why we have replications of fermion families, then we have to assume that the three fermion families transform as a triplet with respect  $S_3$  [2]. This triplet is a reducible representation of  $S_3$ , and more precisely it is the sum of a doublet and a singlet of  $S_3$ . This is a natural choice in the context of third quantization: second quantization is a theory that imposes that all electrons are identical particles, that is the hamiltonian that describes the time evolution of the physical states of  $n$  electrons must be symmetric under permutations of the space coordinates of the  $n$  electron positions. The analogue of this picture in third quantization is a system of  $n$  fermion families that is described by an hamiltonian that is symmetric under permutations  $S_n$  among these fermion families. As a consequence in a world with three families, these families must transform as a triplet of  $S_3$ . In our world this symmetry is not exact, otherwise we would observe two charged leptons degenerate in mass. To spontaneously break this symmetry  $S_3$  we will introduce a scalar field that is a standard model singlet and a triplet with respect  $S_3$ . Also in this case the choice of a  $S_3$  triplet instead of a doublet comes from third quantization. We will assume that the three components of this field  $\phi_i$  (with  $i = 1, 2, 3$ ) take three different vev  $v_3 \gg v_2 \gg v_1$  in order to completely break the group  $S_3$ , while the Higgs responsible for the electroweak breaking is a singlet of  $S_3$ . In fact we will see that even if we choose the electroweak Higgs to be a triplet under  $S_3$ , at the electroweak scale we will have only the singlet component. It is better to require only one Higgs doublet at the electroweak scale to avoid potentially dangerous flavor changing neutral currents.

Let us start observing that operators like

$$\sum_{i,j} \left( g \bar{d}_L^i d_R^j H + g' \bar{d}_L^j d_R^i H \right) \quad (1)$$

that are  $S_3$  invariant are disfavored, because they give two degenerate masses and large mixing angles. In principle one could choose  $g' \gg g$  to obtain one large mass and two light states nearly degenerate, but such a hierarchy is difficult to understand in the context of a grand unified model with  $S_3$  as a flavor group. However this possibility is not ruled out and probably deserves further investigation. We prefer to get rid of these operators, and for this we will make use of an additional  $U(1)$  symmetry from the  $E_6$  group. The  $E_6$  group contains two extra  $U(1)$  that commute with the standard

Fields	$U_{\text{down}}$	$Y$	$I_3^w$	$Q_r$	$Q_t$	Fields	$U_{\text{down}}$	$Y$	$I_3^w$	$Q_r$	$Q_t$
$\bar{D}_1$	+8	$-\frac{1}{3}$	0	2	-2	$e_R^c$	-4	1	0	-1	1
$\bar{D}_2$	+8	$-\frac{1}{3}$	0	2	-2	$\nu_R^c$	-8	0	0	-5	1
$u_{R3}^c$	-4	$-\frac{2}{3}$	0	-1	1	$D_1$	+4	$\frac{1}{3}$	0	-2	-2
$\bar{D}_3$	+8	$-\frac{1}{3}$	0	2	-2	$D_2$	+4	$\frac{1}{3}$	0	-2	-2
$u_{R2}^c$	-4	$-\frac{2}{3}$	0	-1	1	$D_3$	+4	$\frac{1}{3}$	0	-2	-2
$u_{R1}^c$	-4	$-\frac{2}{3}$	0	-1	1	$\bar{N}_L$	+4	$-\frac{1}{2}$	$\frac{1}{2}$	-2	-2
$N_L$	+8	$\frac{1}{2}$	$-\frac{1}{2}$	2	-2	$\bar{E}_L$	+4	$-\frac{1}{2}$	$-\frac{1}{2}$	-2	-2
$d_{L1}$	-4	$\frac{1}{6}$	$-\frac{1}{2}$	-1	1	$X_L$	-12	0	0	0	4
$d_{L2}$	-4	$\frac{1}{6}$	$-\frac{1}{2}$	-1	1	$d_{R1}^c$	0	$\frac{1}{3}$	0	3	1
$d_{L3}$	-4	$\frac{1}{6}$	$-\frac{1}{2}$	-1	1	$d_{R2}^c$	0	$\frac{1}{3}$	0	3	1
$El$	+8	$\frac{1}{2}$	$\frac{1}{2}$	2	-2	$d_{R3}^c$	0	$\frac{1}{3}$	0	3	1
$u_{L1}$	-4	$\frac{1}{6}$	$\frac{1}{2}$	-1	1	$\nu_L$	0	$-\frac{1}{2}$	$\frac{1}{2}$	3	1
$u_{L2}$	-4	$\frac{1}{6}$	$\frac{1}{2}$	-1	1	$e_L$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	3	1
$u_{L3}$	-4	$\frac{1}{6}$	$\frac{1}{2}$	-1	1						

Table 1: Quantum numbers of the irrep **27**

model gauge group. They can be defined from the following embedding  $E6 \supset SO(10) \times U(1)_t \supset SU(5) \times U(1)_r \times U(1)_t$  (see Table 1). If we choose the electroweak Higgs doublet  $H$  with charges  $(q_r, q_t) = (-3, -5)$  the Yukawa couplings in eq.(1) are forbidden. In this case the Yukawa couplings for the quark sector arise as higher mass dimension operators, and only after both the  $S_3$  symmetry and the additional  $U(1)$  are broken (this explains why this symmetry is not observed in the quark masses). To be more specific, we can choose the triplet of  $S_3$ ,  $\phi_i$ , responsible for the  $S_3$  breaking to have the following charges  $(q_r, q_t) = (+5, +7)$ . With these choices, the following operators are compatible both with  $S_3$  and  $U(1)_r \times U(1)_t$

$$\sum_{i,j,k} \left( g \bar{d}_L^i d_R^j H \phi_i + g_L \bar{d}_L^i d_R^j H \phi_i + g_R \bar{d}_L^i d_R^j H \phi_j + g_3 \bar{d}_L^i d_R^j H \phi_k \right). \quad (2)$$

As we already said, when the fields  $\phi_i$  will take a vev, the Yukawa coupling between the electroweak Higgs and the quarks will appear. To go forward with the model building, we will use a statistical analysis where some preferred values for the  $g$  couplings have been selected. An extensive discussion

of the method can be found in [4]. In particular it has been found that  $g_L$  is slightly smaller than  $g$ , while  $g_R$  and  $g_3$  are of the same order of magnitude and very small. Furthermore it has been found  $v_3 \gg v_2 \gg v_1$  and that the up quark mass matrix is approximately diagonal.

up	down
$\phi_1^u/\phi_3^u = m_u/m_t$	$\phi_1^d/\phi_3^d = 0.000077$
$\phi_2^u/\phi_3^u = m_c/m_t$	$\phi_2^d/\phi_3^d = 0.054783$
$g = 1$	$g = 0.848$
$g_L \simeq 0$	$g_L = 0.305 e^{-i 0.02}$
$g_R \simeq 0$	$g_R = 0.002 e^{-i 0.12}$
$g_3 \simeq 0$	$g_3 = 0.009 e^{i 1.82}$

Table 2:  $g$  couplings in the quark sector. We define  $\phi_3 = 1$  in order to set the mass scale of all  $g$  couplings. These have the dimension of the inverse of a mass.

The fact that  $g_3$  and  $g_R$  are very small and of the same order of magnitude can help us in the construction of the model, since this can indirectly indicate the existence of an unbroken symmetry that forbids these operators. In fact we can extend the standard model with few additional U(1) as follows: let us choose the following group  $G = (U(1)_r \times U(1)_t)^3 \rtimes S_3$ . Each  $(U(1)_r \times U(1)_t)$  gauge symmetry factor needs two gauge bosons that belongs to the  $i$ -th family. In practice we enlarge the concept of family to the gauge bosons. Each family contains 27 fermions (the irreducible representation of  $E_6$ ) plus two gauge bosons of the  $(U(1)_r \times U(1)_t)$  group relative to the same family. These two gauge bosons of the  $i$ -th family interact only with the  $i$ -th fermion family<sup>1</sup>. These two gauge bosons transform as a triplet with respect the permutation group  $S_3$ . For this reason the group  $G = (U(1)_r \times U(1)_t)^3 \rtimes S_3$  is the semidirect product and not the direct product. It is easy to see that only the operators below are compatible with G

$$g \bar{d}_L^i d_R^i H^i \phi_i. \quad (3)$$

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<sup>1</sup>Next we will see that each family will contain all the 78 bosons of  $E_6$ , but here we focus on just these two abelian factors  $(U(1)_r \times U(1)_t)$ , since the rest of gauge bosons are irrelevant for the discussion below.

Note that we have added the family index  $i$  also to the electroweak doublet  $H^i$ . This is a necessary choice if we want to make a  $G$  invariant lagrangian. On the contrary, the operator

$$\sum_{i,j} g_L \bar{d}_L^i d_R^j H^i \phi_i \quad (4)$$

is not  $G$  invariant. In fact for  $j \neq i$ , the  $d_R^j$  is the only fermion field that carries non zero charges with respect the  $U(1)$  of  $j$ -th family: the operator (4) would violate the charge conservation of the  $j$ -th family gauge group. This justifies  $g \gg g_L$ . This hierarchy is not large, as we can see in Table 2. A hierarchy much more important is between  $g_L$  and  $g_3, g_R$ . To explain such a hierarchy we can properly choose the breaking direction of  $G$ . The only choice consistent with  $g_L \gg g_R, g_3$  is the breaking of  $G$  in  $G' = (U(1)_{\text{down}})^3 \triangleright S_3$ , where  $U(1)_{\text{down}}$  is the linear combination of  $U(1)_r$  and  $U(1)_t$  that gives a neutral right-handed down quark. To be more precise, if we choose the generator of  $U(1)_{\text{down}}$  to be the linear combination  $Y_r - 3Y_t$  (see Table 1), then only the following operators are  $G'$  invariant

$$g \bar{d}_L^i d_R^i H^i \phi_i + g_L \bar{d}_L^i d_R^j H^i \phi_i \quad (5)$$

In fact the fermionic field  $d_R^j$  does not carry charge with respect  $U(1)_{\text{down}}$ , whatever it is  $j$ . Instead the group symmetry  $G' = (U(1)_{\text{down}})^3 \triangleright S_3$  forbids the operators with couplings  $g_R$  and  $g_3$ . It is at the breaking scale of  $G'$  that also these operators appear. Now let us try to extend the discussion to charged leptons and neutrinos. In  $E_6$ , the charged leptons have the same  $U(1)$  charges of down quarks, once we exchange the left-handed components with the right-handed ones. In fact the  $d_L$  has the same charges of the lepton  $e_R^c$ , while  $d_R^c$  has the same charges of the lepton  $e_L$  (see Table 1). In principle, we could expect that the charged lepton mass matrix is similar in terms of order of magnitude to the down quark mass matrix, except for a transposition operation that exchanges left-handed components with right-handed ones. In  $SU(5)$  unified models, often it happens that the down quark matrix is exactly the transposed of the charged lepton matrix. If we apply such a transposition operation to the down quark mass matrix obtained in the model of Table 2, and we take it as the charged lepton mass matrix, we obtain that the unitary matrix that diagonalizes it is very similar to the  $V_{\text{pmns}}$  mixing matrix that fit all neutrino oscillation data. This result would be very interesting, because

it would provide us with an explanation of neutrino oscillation data and their peculiarities, with no further effort. A similar matrix has been discussed in [7]. However such a scheme of exact SU(5) unification does not work. In fact lepton physical masses are different from the down quark masses at the unification scale, in particular the electron is much lighter than the down quark, and this contradicts the fact that the transposed matrix has the same eigenvalues of the original one. The minimal SU(5) with the exact unification must be discarded, but a relation between the elements of the two matrices (lepton and down quark ones) in terms only of order of magnitudes is still allowed. Namely the relations below

$$\begin{aligned} g_L^d &\simeq g_R^l \\ g_R^d &\simeq g_L^l \\ g_3^d &\simeq g_3^l \end{aligned} \tag{6}$$

are compatible with the charges of the group G. If relations (6) would be exact, we would be back to the exact minimal SU(5) unification, and the electron mass would be too large<sup>2</sup>. To reconcile the electron mass with its experimental value is enough to increase  $g^l$  by a factor of two, with respect  $g_d$  (see Table 3), and this is not incompatible with the group G. The increase of  $g^l$  improves the fit of all charged lepton physical masses, but the mass matrix becomes closer to a diagonal matrix; as a consequence the oscillation angles deduced from it are no longer large enough to explain flavor oscillation in neutrino phenomenology. Namely taking the  $g$  couplings for the charged leptons as in Table 3 we get the following mixing angles

$$\begin{pmatrix} 0.954 & 0.197 & 0.221 \\ 0.160 & 0.947 & 0.276 \\ 0.249 & 0.251 & 0.934 \end{pmatrix}.$$

The explanation for neutrino oscillations will be given in the next section in terms of new symmetries in the neutrino sector.

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<sup>2</sup>Another possibility, not explored here, is to add an abelian factor  $(U(1)_{\text{hyper}})^3$  to the group G, whose generator is proportional to the usual hypercharge. Such an extended G group would contain charges that distinguish leptons and down quarks and would not give relations (6).



down	lepton
$g^d = 0.848$	$g^l = 1.534$
$g_L^d = 0.305 e^{-0.02 i}$	$g_L^l = 0.007 e^{-0.12 i}$
$g_R^d = 0.002 e^{-0.12 i}$	$g_R^l = 0.267 e^{-0.02 i}$
$g_3^d = 0.009 e^{1.82 i}$	$g_3^l = 0.007 e^{1.82 i}$

Table 3:  $g$  couplings in the down quark sector and charged lepton sector. The mass scale has been set as in Table 2.

## Neutrino oscillations

In the previous section we have seen that extending the standard model with an additional gauge symmetry group  $G = (U(1)_r \times U(1)_t)^3 \rtimes S_3$  we are able to give a realistic explanation for masses and mixings in the quark sector. This is also compatible with the physical charged lepton masses. The  $G$  group can be embedded in a larger and unified symmetry group. Furthermore it is not in contradiction with physical quark and lepton masses, differently from the minimal SU(5) predictions<sup>3</sup>.

After the assumption of a family permutation symmetry, as derived from third quantization, we have been lead to add new abelian group factors to the gauge group in order to explain quark mass and mixings, and the right hierarchies among different mass operators.

Let us try to explain large neutrino mixings. In a previous work [5] we have shown that the  $S_3$  symmetry and its breaking into  $S_2$ , can explain the peculiar pattern of the  $V_{\text{pmns}}$ . In fact, in that paper we started from the matrix below

$$m_{\text{maj}} = \begin{pmatrix} a + b + \varepsilon & a & a \\ a & a + b & a \\ a & a & a + b \end{pmatrix} \quad (7)$$

showing that it is diagonalized by the so-called tri-bimaximal matrix in the

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<sup>3</sup>Minimal SU(5) exact unification is also disfavored by the standard model measurement of gauge coupling that do not exactly unify at the unification scale [11].

limit  $a \gg \varepsilon$

$$V_{\text{PMNS}} = \begin{pmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (8)$$

The matrix (7) is  $S_3$  symmetric in the limit  $\varepsilon \ll a, b$ , while the small parameter  $\varepsilon$  breaks  $S_3$  into  $S_2$ . From one side the matrix (7) gives a simple explanation for the mixing angles in the  $V_{\text{pmns}}$ , on the other side it does not give a clear motivation for the mass hierarchies in solar and atmospheric experiments. Additional hypothesis are needed to justify the right hierarchy. In fact the mass matrix (7) has two almost degenerate mass eigenstates, (if  $\varepsilon \ll a, b$ ) both with mass  $b$  and one state with mass  $3a + b$ . Neutrino phenomenology requires that the non degenerate state, that usually is the heaviest one, does not mix with the electron neutrino, and it is maximally mixed with the tau and muon neutrino. This is not consistent with the diagonalization of the matrix (7), where the non degenerate state, with mass  $3a + b$  is the  $S_3$  singlet, thus it has a large mixing with the electron neutrino. Further effort is needed to understand the solar and atmospheric masses. We have to specify the values of  $b, a, \varepsilon$  and understand how to modify their relations with solar/atmospheric mass data. It is easy to realize that we have to take  $b \ll \varepsilon \ll a$  in order to have three masses with different order of magnitudes. But this is not yet enough, because the lightest neutrino is the one with mass  $b$ , that does not mix with the electron neutrino, while it must be the heaviest in order to be compatible with the experiments. To achieve the final result, we also need to make neutrino masses proportional to the inverse of the free parameters  $b, \varepsilon, a$ . Therefore we exploit the seesaw mechanism as follows. Let us imagine that left-handed neutrino masses comes out from the usual seesaw mechanism. In this case the light neutrino mass matrix is proportional to

$$M_L = M_{\text{dirac}} M_{\text{maj}}^{-1} M_{\text{dirac}}^t \quad (9)$$

where  $M_{\text{maj}}$  is the majorana mass of the right-handed neutrino while  $M_{\text{dirac}}$  is the dirac mass between the left-handed and right-handed neutrinos. In eq. (9) the majorana masses  $M_{\text{maj}}$  are typically of the order of the unification scale, while  $M_{\text{dirac}}$  is of the order of the electroweak scale. If  $M_{\text{dirac}}$  is proportional to the identity matrix, the unitary matrix that diagonalizes  $M_L$  is identical to the one that diagonalizes  $M_{\text{maj}}$ , while the eigenvalues of  $M_L$  will be proportional to the inverse of the eigenvalues of  $M_{\text{maj}}$ . Namely,

if  $b \ll \varepsilon \ll a$ , if  $M_{\text{maj}}$  is equal to eq. (7) and if

$$M_{\text{dirac}} = mI \quad (10)$$

where  $I$  is the identity matrix, we get the following physical masses for the lightest neutrinos

$$\begin{aligned} m_1 &= \frac{3m^2}{2\varepsilon} \\ m_2 &= \frac{m^2}{3a} \\ m_3 &= \frac{m^2}{b} \end{aligned} \quad (11)$$

while the diagonalization unitary matrix is the (8). This result is known [5], but now we want to deduce the matrices (7),(10) and the hierarchy  $a \gg \varepsilon \gg b$  from new fundamental symmetries and their spontaneous breaking. Explaining the matrix (10) is easy, since it is enough to extend the previously discussed symmetries in the quark sector (see the previous section) also to neutrinos: observing that the Higgs  $H_i$  of the  $i$ -th family has charges  $(q_r, q_t) = (+3, +5)$  with respect the  $U(1)$  of the  $i$ -th family. In this case the following Yukawa interaction

$$\sum_i g \bar{\nu}_L^i X_L^i H^i \quad (12)$$

is the only Yukawa coupling allowed by the  $G$  symmetry. In fact the fermion  $X_L^i$  is the singlet of  $SU(3) \times SU(2) \times U(1)$  with  $(q_r, q_t) = (0, 4)$  with respect the additional  $U(1)$  of the  $i$ -th family (see Table 1). It is easy to check that  $g \bar{\nu}_L^i X_L^j H_i$  is  $S_3$  invariant but for  $i \neq j$  the  $U(1)$ -charges of the  $i$ -th family are not conserved. Note that the  $G$  symmetry allows for the tree level coupling (12) only for neutrinos. Quark and charged leptons (including up quarks that usually unify with neutrinos in  $SO(10)$ ) need higher mass dimension operators, that is operators including a new field  $\phi_i$  with non zero  $U(1)$  charges to make a gauge invariant term. This is an important property of the model (see [6]). This is to avoid a unification relation between the up quarks and neutrinos, that would make difficult to make consistent mass and mixings in the up quark sector with those observed in the neutrino sector<sup>4</sup>. It is not difficult to reproduce the Dirac mass matrix of eq.(10), but the matrix of eq. (7) and in particular the hierarchy  $b \ll \varepsilon \ll a$ , cannot be deduced in an

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<sup>4</sup>To avoid the unification between up quarks and neutrinos we have added two  $U(1)$  of  $E_6$ . The unique  $U(1)$  that comes from  $SO(10)$  would not be sufficient [6].

obvious way from the model with the  $G$  symmetry that we have considered above. In fact it is not possible to deduce from the  $G$  symmetry that the majorana masses below

$$m_0 X_L^i X_L^i + m_1 X_L^i X_L^j \quad (13)$$

satisfy  $m_0 \ll m_1$  or equivalently  $b \ll a$ . Unfortunately the  $G$  symmetry implies  $m_0 > m_1$ . We can try to enlarge the flavor symmetry  $G$  in order to understand this difference. Third quantization requires that the flavor symmetry is the permutation symmetry of all families [2]. Thus we can enlarge  $S_3$  into  $S_4$ , but we have to introduce a fourth family. This fourth family cannot have the same quantum numbers of the first three families with respect the standard model gauge group, otherwise, we would have seen it.<sup>5</sup> Third quantization and  $S_4$  symmetry offer the possibility to extend the concept of family to all gauge bosons (and not only to the  $U(1)$  as we have done in the previous section), and to avoid the problem of having four chiral families at the electroweak scale. We will show in the following how to proceed. In the previous section we have only mentioned the possibility to consider a grand unified group, we are now able to give an explicit choice. Let us consider the group  $E_6^4 \rtimes S_4$ . We remind that  $\rtimes$  means the semidirect product: the action of the elements of  $S_4$ , onto the gauge group  $E_6^4$  is to permute the different  $E_6$  factors of the full group  $E_6^4$ . In other words the concept of family is extended to the gauge group and the gauge bosons. In this specific case, before the spontaneous symmetry breaking, each family contains 27 fermions (the smallest irreducible representation of  $E_6$ ) and 78 gauge bosons ( $E_6$  gauge bosons). Fermions and bosons of each family interact among them, but do not interact with fermion and bosons of a distinct family. After the spontaneous symmetry breaking of the gauge symmetry, the standard model gauge bosons are a linear combination of the gauge bosons of the first three families. This explains why only the first three families have non zero charges, with respect  $SU(3) \times SU(2) \times U(1)$  of the standard model. For example, if we label  $\lambda_i^a$  the generator of the color  $SU(3)$  of the  $i$ -th family (*i.e.* it acts only on the fermions of the  $i$ -th family) we have that the generator of the QCD group of the standard model, will be given by the linear combination

$$\lambda^a = \lambda_1^a + \lambda_2^a + \lambda_3^a. \quad (14)$$

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<sup>5</sup>Precision tests and direct searches seems to disfavor this possibility. Also the number of light neutrinos measured at LEP, appears to forbid this possibility.

In this case the standard model gluon  $A_a^\mu$ , that is contracted to the generator  $\lambda^a$ , only interacts with the first three families, the generator  $\lambda_4^a$  being missing in the combination (14). It is not difficult to imagine a spontaneous symmetry breaking pattern of the  $E_6^4$  symmetry that gives the generators in eq. (14) as the only unbroken generators of the group  $SU(3)$ . One can easily check that the group  $G$  of the previous section is a subgroup of  $E_6^4 \supset S_4$ , and we understand that the group  $G$  appears at an intermediate scale, as the remaining unbroken symmetry after spontaneous breaking of the full group.

We now proceed to see what happens in the neutrino sector and why such a large unified group helps in understanding the mass hierarchy of the singlet/doublet  $S_3$  components ( $m_0 \ll m_1$  in eq. (13)) and the large splitting between the doublet components. Let us write the most general renormalizable lagrangian, compatible with  $E_6^4 \supset S_4$ , focusing on the neutrino sector (the indices  $ij$  now run from 1 to 4)

$$\sum_{i,j} \left( g \nu_L^i X_L^i H^i + g_1 \nu_R^i X_L^j \omega^{ij} \right). \quad (15)$$

We have added a scalar field  $\omega^{ij}$ , that is a singlet of the standard model. It takes a vev, giving a Dirac mass that mixes  $\nu_R$  and  $X_L$ . The reason of this choice is the following. We have chosen a tensor with two indices  $ij$ , because a field  $\alpha^i$  with only one index would introduce a diagonal mass term  $g_1 \nu_R^i X_L^i \alpha^i$ . This term gives mass both to the doublet and the singlet of  $S_3$ , mixing them. Our aim is different, we want to give mass only to the singlet, leaving the doublets massless. This hierarchy between singlets and doublets is necessary, as we have explained after the example of eq. (13). After having discarded the tensor  $\alpha^i$ , we can consider the two indices tensor  $\omega^{ij}$ : if we take it as a symmetric tensor under the exchange of the  $i$  and  $j$  indices, then this scalar field could take a vev in the direction  $\omega^{ii} = v$  and we obtain a mass both for the singlet and the doublets. There is only one peculiar direction  $\omega^{ij} = v$  (for any choice of  $i$  and  $j$ ) that gives the hierarchy that we need, but we have no fundamental argument to justify this choice.

Instead, if we take an antisymmetric tensor  $\omega^{ij}$  such that  $\omega^{ij} = -\omega^{ji}$ , then the scalar field  $\omega$  no longer has diagonal elements ( $\omega^{ii} = 0$ , by definition of antisymmetric tensor) and as we will see it will give mass only to the singlets of  $S_3$ . A comment is in order here, on the choice of the scalar field  $\omega^{ij}$ : third quantization predicts that if a field belongs to a family it must carry only one index  $i$ . It seems that the scalar field  $\omega^{ij}$  does not satisfy this

rule. However we can imagine that the scalar field  $\omega^{ij}$  is not a fundamental field, but it could be a fermion condensate  $\langle \omega^{ij} \rangle = \langle \chi^i \psi^j \rangle$ . In this case the fundamental fields  $\chi^i$  and  $\psi^j$  carry only one family index as predicted by third quantization. Being an antisymmetric tensor, the scalar field  $\omega^{ij}$  can induce only one renormalizable yukawa interaction, that is the one in eq. (15). It mixes  $\nu_R$  with  $X_L$ , and it cannot mix fields of the same kind ( $X_L^i X_L^j \omega^{ij} = 0$  for the antisymmetric nature of  $\omega$ ). We properly choose the charges of the scalar field  $\omega^{ij}$  in order to make the lagrangian (15)  $E_6^4 \supset S_4$  invariant. The antisymmetric field  $\omega^{ij}$  does not contains  $S_4$  singlet and it must break it. It contains only one  $S_3$  singlet, so it can break  $S_4$  into  $S_3$  as we desired. Namely, if  $S_4$  is broken and the  $S_3$  is unbroken the scalar field  $\omega^{ij}$  can take a vev only in one direction

$$\langle \omega^{ij} \rangle = \begin{pmatrix} 0 & 0 & 0 & v \\ 0 & 0 & 0 & v \\ 0 & 0 & 0 & v \\ -v & -v & -v & 0 \end{pmatrix}_{ij}. \quad (16)$$

It is easy to check that the permutation of the first three families leaves the vev (16) invariant. Putting this vev in eq. (15), we obtain the mass for some neutrino components

$$g\nu_L^i X_L^i H^i + g_1 v \nu_R^i X_L^4 - g_1 v \nu_R^4 X_L^j + \text{h.c.} \quad (17)$$

We get two Dirac neutrinos  $(\nu_R^4, X_L^s)$  and  $(\nu_R^s, X_L^4)$  with mass at the breaking scale of  $S_4$ ,  $g_1 v$ . These two Dirac fermions are very much heavy and are composed by four weyl fermions  $\nu_R^4, X_L^4, X_L^s = X_L^1 + X_L^2 + X_L^3, \nu_R^s = \nu_R^1 + \nu_R^2 + \nu_R^3$ , that transform as singlets under  $S_3$ : all doublets of  $S_3$  remain massless. This accomplishes our task, the same configuration given in the example eq. (7-13), is achieved here: very heavy  $S_3$  singlets compared to very light doublets<sup>6</sup>. The diagonalization of the neutrino mass matrix proceeds similarly to the diagonalization of the simplified model of eq. (7): the main difference is that in the simplified model eq. (7-13) we only have a majorana mass for the three majorana fermions  $X_L^i$ . In this model the dominant mass term mixes  $X_L^i$  and  $\nu_R^i$ , and forms very heavy dirac fermions. In both cases

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<sup>6</sup>All these neutrinos are expected to be much heavier than the weak scale and close to the unification scale.

the unitary matrix that diagonalize the light left-handed neutrinos is given by the tri-bimaximal matrix (8). Deriving the mass eigenvalues is less trivial than the usual seesaw. First we have to give mass to the  $S_3$  doublets that are combinations of  $X_L^i$  and  $\nu_R^i$ . We introduce two scalar fields  $\phi^i$  and  $\chi^i$  (since we have enlarged the permutation group, these are quadruplets under  $S_4$ ). They break  $S_3$  into  $S_2$  through the vev  $\phi^1 = v_1$ ,  $\phi^2 = \phi^3 = v_2$ ,  $\phi^4 = v_4$  and  $\chi^1 = v'_1$ ,  $\chi^2 = \chi^3 = v'_2$ ,  $\chi^4 = v'_4$  with  $v_2 \ll v_1 \ll v$  and  $v'_2 \ll v'_1 \ll v$ . The full lagrangian in the neutrino sector now becomes

$$g\nu_L^i X_L^i H^i + g_1 v \sum_{i=1}^3 \nu_R^i X_L^4 - g_1 v \sum_{j=1}^3 \nu_R^4 X_L^j + g_2 \sum_{i=1}^4 X_L^i X_L^i \phi^i + g_3 \sum_{i=1}^4 \nu_R^i \nu_R^i \chi^i + \text{h.c.} \quad (18)$$

from which we obtain the following masses and mixings for the light left-handed neutrinos when also the Higgs field  $H$  takes a vev <sup>7</sup>  $\langle H_i \rangle = (H, H, H, H_4)$

$$m_1 = \frac{3g^2 H^2}{2g_2 v_1 + g_2 v_2} \quad (19)$$

$$m_2 = \frac{g^2 H^2 v'_4}{3g_1^2 v^2} \quad (20)$$

$$m_3 = \frac{g^2 H^2}{g_2 v_2}. \quad (21)$$

$$(22)$$

These give the following solar and atmospheric masses in the limit  $v'_4 \simeq v \gg v_1 \gg v_2$

$$\Delta m_{\text{atm}}^2 = \frac{g^4 H^4}{g_2^2 v_2^2} \quad (23)$$

$$\Delta m_{\text{sol}}^2 = \frac{9g^4 H^4}{4g_2^2 v_1^2} \quad (24)$$

and the following mixing matrix

$$V_{\text{PMNS}} = \begin{pmatrix} \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (25)$$

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<sup>7</sup>The component  $H_4$  of the field is a standard model singlet and it takes a vev much larger than the electroweak scale, while all the first three components of  $H_i$  take the same vev  $H$ . This is because only the  $S_3$  singlet component of  $H^i$  takes a vev. For an explanation, see the next section.

These results fit the data.

Let us summarize the model. We start with the group  $E_6^4 \supset S_4$ . This group is broken into  $(SU(3) \times SU(2) \times U(1) \times U(1)_r \times U(1)_t)^4 \supset S_4$  at the intermediate scale  $M_X$ . The  $(U(1)_r \times U(1)_t)^4$  factor breaks at the scale  $M'_X$  and we remain with the  $(SU(3) \times SU(2) \times U(1) \times U(1)_{\text{down}})^4 \supset S_4$  symmetry. Where  $U(1)_{\text{down}}$  is the combination of  $U(1)_r$  and  $U(1)_t$  that leaves both the right-handed quark and the left-handed charged leptons neutral. There is another scale  $M''_X$  below the  $M'_X$  at which the group is further broken into  $SU(3) \times SU(2) \times U(1) \times (U(1)_{\text{down}})^3 \supset S_3$ . The last step of breaking is at the scale  $M'''_X$  where we are left with the group  $SU(3) \times SU(2) \times U(1) \times S_3$ . If  $\Lambda$  is the characteristic scale of the fundamental theory  $E_6^4 \supset S_4$  we could expect the following hierarchies for all couplings

$$\begin{aligned}
g_L^d &\simeq \frac{M''_X}{\Lambda^2}; & g_R^l &\simeq \frac{M''_X}{\Lambda^2} \\
g_R^d &\simeq \frac{M'''_X}{\Lambda^2}; & g_L^l &\simeq \frac{M'''_X}{\Lambda^2} \\
g_3^d &\simeq \frac{M'''_X}{\Lambda^2}; & g_3^l &\simeq \frac{M'''_X}{\Lambda^2} \\
g_R^u &\simeq \frac{M'''_X}{\Lambda^2}; & g_L^u &\simeq \frac{M'''_X}{\Lambda^2} \\
g_3^u &\simeq \frac{M'''_X}{\Lambda^2}; & v_2 &\ll v_1 \ll M'''_X
\end{aligned}$$

that are in good agreement with the values derived from the experimental data (see Table 2 and 3)).

## The electroweak symmetry breaking and the flavor symmetry $S_3$

In the previous sections we have seen that the Higgs doublet responsible for the electroweak symmetry breaking has to be chosen in the triplet reducible representation of  $S_3$  in order to make the lagrangian (3) gauge invariant under  $(U(1)_{\text{down}})^3 \supset S_3$ . At the same time we want that there is only one Higgs doublet at the electroweak scale. This is to avoid dangerous flavor changing neutral currents due to the virtual exchange of additional Higgs fields. In this section we will see that requiring just one Higgs doublet at the weak scale implies that only one component of the original triplet  $H^i$  (with



$i = 1, 3$ ) can take a vev, and this component must be a singlet of  $S_3$ . This is true even if the breaking of  $S_3$  occurs at a much higher scale. The reason is the following.

We can split the effective potential of the three Higgs doublets  $H^i$  as follows

$$V(H^i) = V_{S_3}(H^i) + \Delta V(H^i, \phi^i) \quad (26)$$

where  $V_{S_3}$  is an invariant potential under  $S_3$  while  $\Delta V$  breaks  $S_3$  because it contains all scalar fields  $\phi^i$  that break the  $S_3$  symmetry.  $\Delta V$  is defined in such a way that  $\Delta V(H^i, \phi^i) = 0$  in the limit  $\phi^i = 0$ . Splitting the potential in a  $S_3$  symmetric part plus a  $S_3$  not symmetric part is always possible. The Higgs mass matrix  $M_{kj}$ , ( $k, j$  are family indices) is given by

$$M_{kj} = \frac{\partial^2}{\partial H_k \partial H_j} V(H_i). \quad (27)$$

If we only consider terms coming from  $V_{S_3}(H^i)$ , the mass matrix  $M_{kj}$  will be  $S_3$  invariant. Thus we expect two degenerate eigenstates with mass  $m_D$  that transform as a doublet of  $S_3$ , plus one eigenstate with mass  $m_S$  that is a  $S_3$  singlet. We remind that we require just one light electroweak Higgs. With this assumption we must have  $|m_S^2| \ll |m_D^2|$ . Furthermore only the  $S_3$  singlet component of  $H^i$  can take a vev. In fact a possible vev of the doublet component would be proportional to  $m_D$ . But, at the same time, we know that  $|m_D^2| \gg M_W^2$ , and therefore the doublet cannot take a vev.

Now it is easy to understand that, since  $\Delta V(H^i, \phi^i)$  is just a perturbation of the  $S_3$  symmetric part, the minimization of the full potential  $V(H_i)$  can only give a very tiny mixing between the doublet and singlet components of  $H^i$ , being<sup>8</sup>  $m_D \gg \phi^i$ . This argument does not show in a rigorous way that the Higgs doublet must be the  $S_3$  singlet, but show that is a realistic and reasonable hypothesis [13].

## Conclusions

The experimental mass measurements in neutrino oscillations, together with masses and mixings of the rest of matter fermions, give us an almost complete picture of the less understood sector of the standard model, that is the

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<sup>8</sup> $m_D$  is expected to be not much smaller than the unification scale while  $\phi^i$  is the breaking scale of  $S_3$ .

sector of Yukawa interactions. We also know that grand unification can explain charge quantization, and can embed all fermions of each family in just one irreducible representation of  $SO(10)$ . The simplest and minimal  $SO(10)$  model predicts that the up quark mass matrix is equal to the dirac neutrino matrix. Minimal  $SU(5)$  predicts that the mass matrix of charged leptons is the transposed of the down quark mass matrix. Both predictions are not in agreement with the experimental observation. An effort to go beyond the minimal unification scenario is needed. We also know, that if the fundamental lagrangian of interactions comes from the third quantization, the flavor symmetry group is very likely to be the permutation symmetry. This symmetry group gives good predictions of neutrino oscillations. How to extend such a symmetry to the quark sector in the context of a grand unified group is the subject of this work. In particular we improve some aspects of the model studied in [5, 6] clarifying why in eq.(7)  $a \gg b$ . We have found that the unification group  $E_6^4 \rtimes S_4$ , with  $S_4$  the flavor group and  $E_6^4$  the gauge group, predicts at least one breaking pattern that simultaneously explain masses and mixings of quarks, leptons and neutrinos. It is important to note that the symmetry group is the semidirect product (and not the direct product) between the gauge group and the flavor group. This can descend in a natural way from third quantization and predicts why operators (or interactions) that mix different families (see the discussion after eq.(2)) are smaller than those that do not mix families. We have put in evidence that  $U(1)_{\text{down}}^4 \rtimes S_4$ , that is a subgroup of  $E_6^4 \rtimes S_4$ , predicts the right hierarchies among the couplings of the operators needed to explain quark mass and mixings. We have found that the introduction of the scalar field  $\omega_{ij}$ , antisymmetric with respect  $i$  and  $j$  is useful. Because it explains why the  $S_3$  singlet components of  $X_L$  and  $\nu_R$  are much heavier than the doublets. As a consequence we also get  $\Delta M_{\text{atm}}^2 \gg \Delta M_{\text{sol}}^2$ . This hierarchy was assumed as a starting hypothesis in [5], now a clear motivation for this choice has been given.

Finally we know that the possibility of exact  $SU(5)$  unification in the yukawa sector is excluded, but we have shown that partial unification, *i.e.* in terms of order of magnitudes is still possible. Namely, in our model it exists an intermediate scale, at which the  $U(1)$  factors contained in  $E_6^4$  are unbroken. This abelian group implies that each mass operator of the down quarks is of the same order of magnitude of its analogue in the charged lepton, taking into account that right-handed quarks are partners of left-handed charged leptons. In particular we have found that a factor two in

the relative size between quarks and lepton operators is enough to obtain the electron and down quark mass ratio, and neutrino oscillations in agreement with data.

To understand these small differences in the relative size between quark/lepton operators we need a deeper analysis of the symmetry breaking pattern of  $E_6^4 \rtimes S_4$ .

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